PERTH MODERN SCHOOL
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Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS SPECIALIST
UNIT 1

Section Two:
Calculator-assumed

Your Name: SOLUTIONS

Your Teacher's Name:

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Mark	Max
8		9	15		8
9		4	16		7
10		6	17		10
11		8	18		10
12		10	19		11
13		7			
14		9			

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	46	35
Section Two: Calculator-assumed	12	12	100	100	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 11 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **(100 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

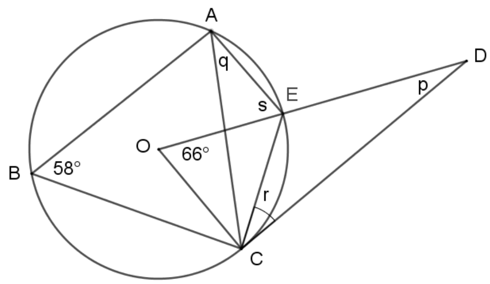
Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 8 {1.3.7, 1.3.9, 1.3.11, 1,3,15} **(9 marks)**

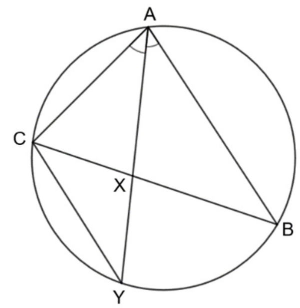
Point O is the centre of the circle below, and $\overline{CD}$ is tangent to the circle.
For each of the angle sizes indicated below, fill in the table.



	Size	Reason
p	24° ✓	$\angle OCD = 90°$ (tangent and radius are perpendicular) $p = 90 - 66 = 24°$ (angles in a triangle add to 180°) ✓ calculates using $\angle OCD = 90°$
q	33° ✓	$\angle EAC = \frac{1}{2}\angle EOC$ (angle at centre is twice angle at circumference) ✓
r	33° ✓	$\angle EAC = \angle ECD$ (alternate segment theorem) ✓
s	65° ✓	$\angle OEC = p + r = 57°$ (exterior angle theorem) ✓ calculates $\angle OEC = 57°$ $\angle AEC = 180 - 58° = 122°$ (opposite angles in a cyclic quadrilateral are supplementary) $\angle AEO = 122 - 57 = 65°$ (adjacent angles) ✓ uses cyclic quad. theorem

Question 9 {1.3.8, 1.3.12} **(4 marks)**

In the diagram on the right, $\overline{AY}$ bisects $\angle BAC$.



a) Prove that ΔABX is similar to ΔCYX. (2 marks)

$AX \cdot XY = BX \cdot XC$ (intersecting chords theorem)

$\frac{AX}{XC} = \frac{BX}{XY}$

$\angle AXB = \angle CXY$ (vertically opposite angles are equal)

$\Delta ABX \mathbin{|||} \Delta CYX$ (~SAS)

✓ uses intersecting chords theorem

✓ proves $\Delta ABX \mathbin{|||} \Delta CYX$

b) Hence, prove that ΔAYC is similar to ΔCYX. (2 marks)

$\angle BAX = \angle YAC$ (Given)

$\angle ABX = \angle AYC$ (angles in same segment are equal)

$\Delta ABX \mathbin{|||} \Delta AYC$ (~AA)

$\Delta ABX \mathbin{|||} \Delta CYX$ (part a)

$\therefore \Delta AYC \mathbin{|||} \Delta CYX$ (both are similar to ΔABX)

✓ proves $\Delta ABX \mathbin{|||} \Delta AYC$

✓ concludes $\Delta AYC \mathbin{|||} \Delta CYX$

Question 10 {1.2.14} **(6 marks)**

At 8pm, a lighthouse detects a ship at a position of $4\boldsymbol{i} - 7\boldsymbol{j}$ km.

(a) If the ship is travelling at $\boldsymbol{i} + 2\boldsymbol{j}$ km/h, express its position t hours after 8pm. (1 mark)

$$\overrightarrow{OS} = 4\boldsymbol{i} - 7\boldsymbol{j} + t(\boldsymbol{i} + 2\boldsymbol{j})$$
$$= (4 + t)\boldsymbol{i} + (-7 + 2t)\boldsymbol{j} \text{ km}$$ ✓correct position

(b) At what time is the ship closest to the lighthouse? (3 marks)

$$\left|\overrightarrow{OS}\right| = \sqrt{(4 + t)^2 + (-7 + 2t)^2}$$
$$= \sqrt{16 + 8t + t^2 + 49 - 28t + 4t^2}$$
$$= \sqrt{5t^2 - 20t + 64}$$ ✓correct magnitude (no need to simplify)

Minimum at $t = -\frac{b}{2a}$:

$$t = -\frac{-20}{2(5)}$$
$$= 2 \text{ hours}$$ ✓ correct number of hours

The ship is closest at 10pm. ✓ correct time

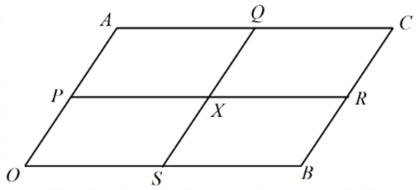
(c) Hence, what is the closest distance the ship is from the lighthouse? (2 marks)

$$\left|\overrightarrow{OS}\right| = \sqrt{5t^2 - 20t + 64}$$
$$= \sqrt{5(2)^2 - 20(2) + 64}$$ ✓ substitutes t=2
$$= \sqrt{20 - 40 + 64}$$
$$= \sqrt{44}$$
$$= 2\sqrt{11}$$

The closest distance is $2\sqrt{11}$ km. ✓evaluates

Question 10 **(8 marks)**

Let $OACB$ be a parallelogram, and let $\boldsymbol{a} = \overrightarrow{OA}$ and $\boldsymbol{b} = \overrightarrow{OB}$. Let P, Q, R and S be the midpoints of $\overline{OA}$, $\overline{AC}$, $\overline{CB}$ and $\overline{OB}$ respectively, and let X be the point of intersection of $\overline{PR}$ and $\overline{QS}$.



(a) Let M be the midpoint of $\overline{SQ}$. **Show** that $\overrightarrow{OM} = \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$. (2 marks)

$\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OB} + \frac{1}{2}\overrightarrow{SQ}$ where $\overrightarrow{SQ} = -\frac{1}{2}\underset{\sim}{b} + \underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$

$= \underset{\sim}{a}$

✓ writes $\overrightarrow{OM}$ as $\frac{1}{2}\overrightarrow{OB} + \frac{1}{2}\overrightarrow{SQ}$ or as $\frac{1}{2}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{PR}$

Note: not sufficient to assume $\overrightarrow{OM} = \overrightarrow{OP} + \overrightarrow{OS}$ without justification using vectors

$\therefore \overrightarrow{OM} = \frac{1}{2}\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{a}$ ✓

(b) Let N be the midpoint of $\overline{PR}$. Determine an expression, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$, for $\overrightarrow{ON}$. (1 mark)

$\overrightarrow{ON} = \frac{1}{2}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{PR}$ where $\overrightarrow{PR} = -\frac{1}{2}\underset{\sim}{a} + \underset{\sim}{b} + \frac{1}{2}\underset{\sim}{a}$

$= \underset{\sim}{b}$

$\therefore \overrightarrow{ON} = \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$ ✓ correct expression

(c) Explain why $\overrightarrow{OX} = \overrightarrow{OM}$. (2 marks)

Since $\overrightarrow{ON} = \overrightarrow{OM} = \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$, $M = N$, and so M is the point of intersection of $\overline{PR}$ and $\overline{QS}$, which is X by definition.

It follows that $\overrightarrow{OX} = \overrightarrow{OM}$.

✓ notes that $\overrightarrow{ON} = \overrightarrow{OM}$

✓ notes that M is point of intersection

(d) Hence, prove that $OPXS$ is a parallelogram. (3 marks)

$\overrightarrow{OS} = \frac{1}{2}\underset{\sim}{b}$ and $\overrightarrow{PX} = \overrightarrow{OX} - \overrightarrow{OP}$

$= \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$

$= \frac{1}{2}\underset{\sim}{b}$.

✓ justifies that $\overrightarrow{PX} = \frac{1}{2}\underset{\sim}{b}$ or $\overrightarrow{SX} = \frac{1}{2}\underset{\sim}{a}$ (or states above with justification given in earlier part of question)

Hence $\overrightarrow{OS} = \overrightarrow{PX}$, which implies that $|\overrightarrow{OS}| = |\overrightarrow{PX}|$ and $\overrightarrow{OS} \parallel \overrightarrow{PX}$.

✓ shows vector are equal

It follows that $OPXS$ is a parallelogram. ✓ concludes parallelogram.

Question 12 {1.2.11, 1.2.12} **(10 marks)**

A circle has centre O at the origin. Point A on the circle has position vector $\overrightarrow{OA} = a\boldsymbol{i} + b\boldsymbol{j}$, and point B on the circle has position vector $\overrightarrow{OB} = b\boldsymbol{i} - a\boldsymbol{j}$, where a and b are real constants. Point C (outside the circle) has position vector $\overrightarrow{OC} = 34\boldsymbol{i} + 14\boldsymbol{j}$.

(a) Determine expressions for each of the vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$. (3 marks)

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 34\underset{\sim}{i} + 14\underset{\sim}{j} - (a\underset{\sim}{i} + b\underset{\sim}{j}) = (34-a)\underset{\sim}{i} + (14-b)\underset{\sim}{j}$$

✓ correct vector

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = 34\underset{\sim}{i} + 14\underset{\sim}{j} - (b\underset{\sim}{i} - a\underset{\sim}{j}) = (34-b)\underset{\sim}{i} + (14+a)\underset{\sim}{j}$$

✓ correct vector

✓ shows calculation for at least one vector

(b) Hence determine expressions for each of $\overrightarrow{AC} \cdot \overrightarrow{OA}$ and $\overrightarrow{BC} \cdot \overrightarrow{OB}$. (3 marks)

$$\overrightarrow{AC} \cdot \overrightarrow{OA} = \left[(34-a)\underset{\sim}{i} + (14-b)\underset{\sim}{j}\right] \cdot (a\underset{\sim}{i} + b\underset{\sim}{j}) = 34a - a^2 + 14b - b^2$$

✓ correct expression

$$\overrightarrow{BC} \cdot \overrightarrow{OB} = \left[(34-b)\underset{\sim}{i} + (14+a)\underset{\sim}{j}\right] \cdot (b\underset{\sim}{i} - a\underset{\sim}{j}) = 34b - b^2 - 14a - a^2$$

✓ correct expression

✓ shows calculation for at least one expression

(c) Given that $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$ are both tangent to the circle, and the radius of the circle is 26, determine the values of a and b. (4 marks)

$\overrightarrow{AC} \cdot \overrightarrow{OA} = 0 \Rightarrow 34a + 14b = a^2 + b^2$

$\overrightarrow{BC} \cdot \overrightarrow{OB} = 0 \Rightarrow 34b - 14a = a^2 + b^2$

✓ equates dot products to 0

Since the circle has radius 26, $|\overrightarrow{OA}| = 26$.

So $|\overrightarrow{OA}|^2 = a^2 + b^2 = 26^2$.

✓ uses $a^2 + b^2 = 26$

Hence $34a + 14b = 26^2$

and $34b - 14a = 26^2$

✓ obtains 2 correct linear equations in a & b

Solving simultaneously gives $a = 10$ and $b = 24$

✓ obtains correct values for a & b

Question 13 {1.1.1-1.1.4} **(7 marks)**

The 10 letters of the word

O P A P O P A P O P

are written on 10 separate pieces of card. These cards are arranged in a line next to each other.

(a) How many different word arrangements (of length 10) can be made? (2 marks)

$$\frac{10!}{5!3!2!} = 2520$$

✓ Numerator
✓ denominator

(b) How many arrangements start and end with P? (2 marks)

$$\frac{8!}{3!3!2!} = 2520$$

✓ Numerator
✓ denominator

(c) The ten letters are arranged in a random order. Determine the probability that the resulting arrangement will start and finish with the same letter.

(3 marks)

$$\frac{\frac{8!}{3!3!2!} + \frac{8!}{5!2!} + \frac{8!}{5!3!}}{\frac{10!}{5!3!2!}} = \frac{784}{2520} = \frac{14}{45}$$

✓ Setting up addition of permutations
✓ Numerator
✓ denominator

Question 14 {1.1.1-1.1.5} **(9 marks)**

The nine single digit numbers are written on nine separate pieces of card.

1, 2, 3, 4, 5, 6, 7, 8, 9

Four of these cards are picked at random and placed next to each other to form a four digit number. Determine the probability that the four-digit number will be formed with the given conditions:

(a) has both odd and even digits. (3 marks)

total different arrangments: ${}^{9}\mathbf{P}_4 = 3024$

all even numbers: ${}^{4}\mathbf{P}_4 = 24$

all odd numbers: ${}^{5}\mathbf{P}_4 = 120$

$$\therefore \text{ has odd and even digits: } \frac{3024 - (24 + 120)}{3024} = \frac{20}{21}$$

- ✓ determining total number of all even and all odd numbers
- ✓ Numerator
- ✓ Denominator

(b) the number has at least three odd digits. (3 marks)

all odd: 120

3 odd numbers: ${}^{5}\mathbf{P}_3 \times 4 \times 4 = 960$

$$P(\text{at least 3 odd digits}) = \frac{120 + 960}{3024} = \frac{5}{14}$$

- ✓ determining total number of all odd numbers and at least 3 odd digits
- ✓ Numerator
- ✓ Denominator

(c) the sum of the four digits is 28.

(3 marks)

$$P(\text{ sum of four digits is 28}) = \frac{4! \times 2}{3024} = \frac{1}{63}$$

- ✓ Recognising that there are 2 groups of combinations of numbers that make 28, hence x2.
- ✓ Numerator
- ✓ Denominator

Question 15 {1.1.6, 1.1.7, 1.1.8} **(8 marks)**

(a) How many 8-character passwords can be generated using the symbols ***!@#$%^&****

i) if the first three symbols must be ***$&@*** (in any order) and the next two symbols must be ***^**** (in any order)? (2 marks)

Solution
$3! \times 2! \times 3! = 72$
Specific behaviours
✓ uses multiplicative reasoning
✓ correct number

ii) if the order of ***!#%&*** cannot be changed, but their placement may be changed? (e.g. ***!#%&***@$^* and @$***!#***^****%&*** are acceptable, but ***&#%!***@$^&* and @$***%!***^****&#*** are not) (3 marks)

Solution
$\binom{8}{4} \times 4! = 1680$
Specific behaviours
✓ correct selections of placement for !#%&
✓ correct arrangements of placement for @$^*
✓ correct number

(b) Year 7 students at Perth Modern School were asked to write a password generator in Python with a mix of lowercase letters, uppercase letters, whole numbers, and symbols from (a). Each password must have length six. **Prove** that at least one character will be used more than once within 12 generated passwords. (3 marks)

Solution
12 passwords will require $12 \times 6 = 72$ characters (pigeons). The number of characters available is $26 \times 2 + 10 + 8 = 70$ (pigeonholes). Since the number of characters used exceeds the number of characters available, the Pigeonhole Principle guarantees that among 12 passwords at least one character will be used more than once.
Specific behaviours
✓ identify pigeonholes
✓ identify pigeons
✓ uses the Pigeonhole Principle

Question 16 {1.1.7, 1.1.8} **(9 marks)**

A team of six students is to be selected for a Mathematics competition from ten Year 11 and eight Year 12 students. How many different teams are possible if:

a) there are no restrictions. (2 marks)

Number of teams $= \binom{18}{6}$ ✓

$= 18564$ ✓

b) there must be exactly 3 Year 11 students and 3 Year 12 students. (2 marks)

Number of teams $= \binom{10}{3} \times \binom{8}{3}$ ✓ multiplies

$= 6720$ ✓

c) there must be at least 2 Year 11 students and at least 2 Year 12 students. (3 marks)

Number of teams $= \binom{10}{2}\times\binom{8}{4} + \binom{10}{3}\times\binom{8}{3} + \binom{10}{4}\times\binom{8}{2}$ ✓ adds numbers corresponding to different compositions

$= 15750$ ✓✓

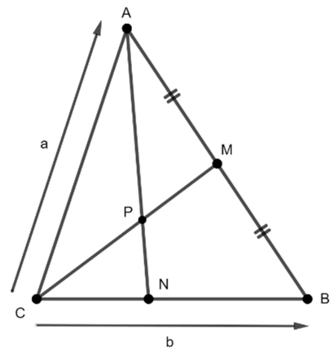
d) one student in the team is assigned to the role of captain, and another student to the role of vice-captain, and there are no other restrictions. (Note that teams consisting of the same students count as distinct if the roles of captain and/or vice-captain are assigned differently). (2 marks)

Number of teams $= \binom{18}{1}\times\binom{17}{1}\times\binom{16}{4}$ ✓

$= 556920$ ✓

Question 17 {1.2.1 – 1.2.5} **(10 marks)**

a) In the diagram below, M is the midpoint of AB, $\boldsymbol{a} = \overrightarrow{CA}$ and $\boldsymbol{b} = \overrightarrow{CB}$. Given that $CP : PM = 3 : 2$, determine the value for λ if $\overrightarrow{CN} = \lambda\overrightarrow{CB}$. (5 marks)



Solution
$\overrightarrow{AB} = -\boldsymbol{a} + \boldsymbol{b}$
$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB} = -\frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$
$\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \boldsymbol{a} - \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b} = \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$
$\overrightarrow{CP} = \frac{3}{5}\overrightarrow{CM} = \frac{3}{10}\boldsymbol{a} + \frac{3}{10}\boldsymbol{b}$
$\overrightarrow{AP} = \overrightarrow{AC} + \overrightarrow{CP} = -\boldsymbol{a} + \frac{3}{10}\boldsymbol{a} + \frac{3}{10}\boldsymbol{b} = -\frac{7}{10}\boldsymbol{a} + \frac{3}{10}\boldsymbol{b}$
$\overrightarrow{AN} = \overrightarrow{AC} + \overrightarrow{CN} = -\boldsymbol{a} + \lambda\boldsymbol{b}$
$\overrightarrow{AN}$ *is a scalar multiple of* $\overrightarrow{AP}$ *since in the same direction*
$\because \left(-\frac{7}{10}\right)\left(\frac{10}{7}\right) = -1$
$\therefore\ \lambda = \left(\frac{3}{10}\right)\left(\frac{10}{7}\right) = \frac{3}{7}$
Specific behaviours
✓ Expresses $\overrightarrow{AP}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ ✓ Expresses $\overrightarrow{AN}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ ✓ Uses $\overrightarrow{AN}$ is a scalar multiple of $\overrightarrow{AP}$ ✓ Finds the scalar multiple $\frac{10}{7}$ ✓ Correct value for λ

b) Let $\boldsymbol{c} = \begin{bmatrix} \boldsymbol{k-1} \\ \boldsymbol{6} \end{bmatrix}$ and $\boldsymbol{d} = \begin{bmatrix} \boldsymbol{-1} \\ \boldsymbol{k+1} \end{bmatrix}$. Find the value of k, if $|\boldsymbol{d} - \boldsymbol{c}| = \frac{k(\boldsymbol{c}\cdot\boldsymbol{d})}{5k+7}$. (5 marks)

Solution
$\boldsymbol{d} - \boldsymbol{c} = \begin{bmatrix} -1-(k-1) \\ k+1-6 \end{bmatrix} = \begin{bmatrix} -k \\ k-5 \end{bmatrix}$
$\lvert\boldsymbol{d} - \boldsymbol{c}\rvert = \sqrt{(-k)^2 + (k-5)^2}$
$= \sqrt{2k^2 - 10k + 25}$
$\boldsymbol{c}\cdot\boldsymbol{d} = -(k-1) + 6(k+1) = 5k + 7$
$\sqrt{2k^2 - 10k + 25} = \frac{k(5k+7)}{5k+7}$
$2k^2 - 10k + 25 = k^2$
$k^2 - 10k + 25 = 0$
$k = 5$
Specific behaviours
✓ determines $\boldsymbol{d} - \boldsymbol{c}$ ✓ determines $\lvert\boldsymbol{d} - \boldsymbol{c}\rvert$ ✓ determines $\boldsymbol{c}\cdot\boldsymbol{d}$ ✓ writes equation for k ✓ correct value for k

Question 18 {1.2.6 – 1.2.9} **(9 marks)**

Elizabeth Quay Jetty (Point E) and Mends St Jetty (Point M) are such that $\overrightarrow{EM} = (-335\boldsymbol{i} - 1287\boldsymbol{j})$ m. A yacht is to be sailed from E to M. In still water, the yacht can maintain a steady speed of 3.2 m/s. The wind is blowing with a steady velocity $3\boldsymbol{i} - \boldsymbol{j}$.

(a) Find, in the form $a\boldsymbol{i} + b\boldsymbol{j}$, the velocity vector the sailor should set so that the yacht can travel from E to M. (7 marks)

Solution
$\boldsymbol{v}_r = (a+3)\boldsymbol{i} + (b-1)\boldsymbol{j}$
Also, $v_r = \lambda\overrightarrow{EM} = -335\lambda\boldsymbol{i} - 1287\lambda\boldsymbol{j}$
$(a+3)\boldsymbol{i} + (b-1)\boldsymbol{j} = -335\lambda\boldsymbol{i} - 1287\lambda\boldsymbol{j}$
$\begin{cases} a+3 = -335\lambda \\ b-1 = -1287\lambda \end{cases}$
$b = \frac{1287}{335}a + \frac{4196}{335}$
$\sqrt{a^2+b^2} = 3.2$
$a^2 + (\frac{1287}{335}a + \frac{4196}{335})^2 = 3.2^2$
$a = -3.19 \text{ and } b = 0.28$
$a = -2.92 \text{ and } b = 1.32$
Since $\lambda > 0 \Rightarrow a+3 < 0 \;\&\; b-1 < 0$
Hence, $a\boldsymbol{i} + b\boldsymbol{j} = -3.19\boldsymbol{i} + 0.28\boldsymbol{j}$
Specific behaviours
✓ Expresses resultant velocity of the yacht as the sum of velocities of the yacht and the wind ✓ Expresses resultant velocity of the yacht as a scalar multiple of $\overrightarrow{EM}$ ✓ Writes simultaneous equations for a and b ✓ Expresses the magnitude of velocity of yacht in terms of a and b ✓ Writes a quadratic equation for a or b ✓ Correct values for a or b ✓ Chooses correct set of values for a and b

(b) Determine, to the nearest minute, the time the yacht takes to travel from Elizabeth Quay Jetty to Mends St Jetty. (2 marks)

Solution
$t = \frac{-335}{-3.19+3}\text{m/s} =$ $\frac{1763}{60} = 29 \text{ minutes}$
Specific behaviours
✓ Finds the magnitudes of $\overrightarrow{EM}$ and resultant velocity ✓ Correct time taken (accept 1783s/30min if unrounded values used)

Question 19 {1.2.10 – 1.2.13} **(11 marks)**

If $\boldsymbol{r} = 2\boldsymbol{i} - \boldsymbol{j}$, $\boldsymbol{s} = x\boldsymbol{i} - 4\boldsymbol{j}$, $\boldsymbol{t} = -3\boldsymbol{i} + 4\boldsymbol{j}$, and $\boldsymbol{u} = \boldsymbol{i} - 8\boldsymbol{j}$ determine

(a) the value of x such that $\boldsymbol{r} \perp \boldsymbol{s}$. (2 mark)

Solution
$2x + (-1) \times (-4) = 0$ $x = -2$
Specific behaviours
✓ State the scalar product is 0   ✓ Correct value for x

(b) the angle between the directions of $\boldsymbol{s}$ and $\boldsymbol{t}$ to the nearest tenth of a degree (using the value of x found in part (a)). (2 marks)

Solution
$\theta = \cos^{-1}\left(\frac{\boldsymbol{s} \cdot \boldsymbol{t}}{\lvert\boldsymbol{s}\rvert\lvert\boldsymbol{t}\rvert}\right) = \cos^{-1}\left(\frac{(-2) \times (-3) + (-4) \times 4}{2\sqrt{5} \times 5}\right)$ $= 116.6°$
Specific behaviours
✓ correct expression for scalar product with cos   ✓ states angle

(c) the unit vector in the direction of $\boldsymbol{u}$. (2 marks)

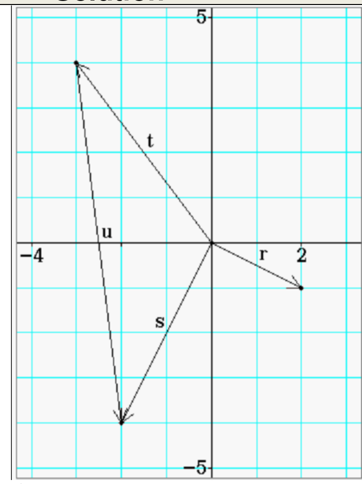
Solution
$\hat{\boldsymbol{u}} = \frac{\boldsymbol{u}}{\lvert\boldsymbol{u}\rvert} = \frac{\boldsymbol{i} - \boldsymbol{8j}}{\sqrt{1^2 + (-8)^2}} = \frac{1}{\sqrt{65}}(\boldsymbol{i} - \boldsymbol{8j})$
Specific behaviours
✓ correct magnitude   ✓ correct unit vector in component form

(d) the vector projection (i.e. the vector resolute) of $\boldsymbol{t}$ on $\boldsymbol{u}$. (2 marks)

Solution
$(\boldsymbol{t} \cdot \hat{\boldsymbol{u}})\hat{\boldsymbol{u}} = [-3 \times \frac{1}{\sqrt{65}} + 4 \times (-\frac{8}{\sqrt{65}})]\frac{1}{\sqrt{65}}(\boldsymbol{i} - \boldsymbol{8j})$ $= -\frac{7}{13}\boldsymbol{i} + \frac{56}{13}\boldsymbol{j}$
Specific behaviours
✓ correct expression for projection vector   ✓ correct projection vector in component form

(e) the area of triangle that vectors $\boldsymbol{s}$, $\boldsymbol{t}$, and $\boldsymbol{u}$ form by drawing a diagram with all the given vectors (3 marks)

Solution



$$Area = \frac{1}{2} \times 2\sqrt{5} \times 5 \times \sin 116.6° = 10\ units^2$$

Specific behaviours

✓ correct diagram with given vectors
✓ correct expression for area of triangle
✓ correct value

Additional working space

Question number: _______

Additional working space

Question number: ________

Additional working space

Question number: ________